

Discrete Math

Venn Diagrams and Area of Shapes

Grades 4-6

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Executive Summary: We have included two main units for students in grades 4-6: Venn Diagrams and Using Discrete Mathematics to Find the Area of Shapes.

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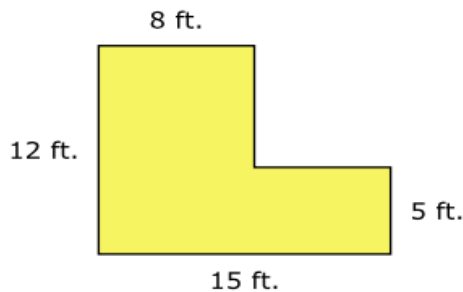
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MN K-12 Academic Standards in Mathematics:

- **4.3.2.3** Understand that the area of a two-dimensional figure can be found by counting the total number of same size square units that cover a shape without gaps or overlaps. Justify why length and width are multiplied to find the area of a rectangle by breaking the rectangle into one more unit by one unit squares and viewing these as grouped into rows and columns.



What is the area of the floor?

- A. 40 sq. ft.
 - B. 131 sq. ft.
 - C. 171 sq. ft.
 - D. 180 sq. ft.
- **4.3.2.4** Find the areas of geometric figures and real-world objects that can be divided into rectangular shape. Use square units to label measurements.

Kira is using 1-inch square tiles to cover a table top. The table top is 24 inches long and 18 inches wide. She lays the tiles into strips of 6.



How many strips of tiles will Kira need to cover the table with no gaps or overlaps?

- A. 14
 - B. 18
 - C. 72
 - D. 432
- **5.3.2.1** Develop and use formulas to determine the area of triangles, parallelograms and figures that can be decomposed into triangles.

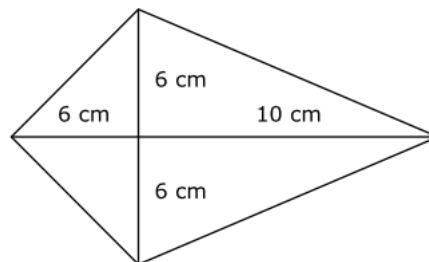
A triangle has a height of 25 feet. The length of its base is 12 feet. What is the area of the triangle?

Enter your answer in the box.

square feet

- **6.3.1.2** Calculate the perimeter, area, surface area and volume of two-and three-dimensional figures to solve real-world and mathematical problems.

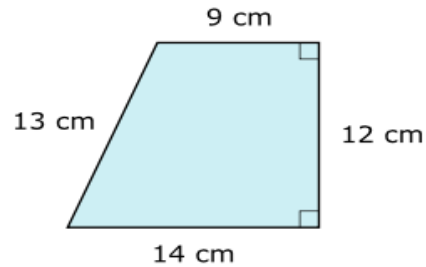
A scale drawing of a kite is shown.



What is the area of the kite?

- A. 28 cm^2
- B. 60 cm^2
- C. 96 cm^2
- D. 192 cm^2

A trapezoid is shown.



What is the area of the trapezoid?

- A. 48 cm^2
- B. 138 cm^2
- C. 168 cm^2
- D. 173 cm^2

6.1.1.6 Determine greatest common factors and least common multiples. Use common factors and common multiples to calculate with fractions and find equivalent fractions.

19. What is the greatest common factor of 48 and 64?

- A. 2
- B. 8
- C. 16
- D. 24

Unit 1: Venn Diagrams

Days 1-2: Classifying Polygons

(Taken from Learnzillion.com)

[Pre-Test and Post-Test](#)

Materials: Shapes, paper, pencil

Launch: “Yesterday I was organizing my bin of buttons, and I couldn’t decide how to sort them. I had red, yellow, and green colored- round, square, and oval buttons. What are some ways I can sort these buttons?”

“Amy’s teacher gave her some shapes for her investigation. As she observes these shapes, she notices that they exhibit some differences and similarities.”

How can you describe these shapes based on their attributes?

How are these shapes the same?

How are these shapes different?
Can you think of another way to sort these figures?

Launch

Amy's teacher gave her some shapes for her investigation. As she observes these shapes, she notices that they exhibit some differences and similarities.



Do you notice any differences or similarities? Explain your thinking.

Explore:

What is the task asking you to do?
What information do you have?
Is there another way to think about this problem?
Explain why your solution is reasonable.

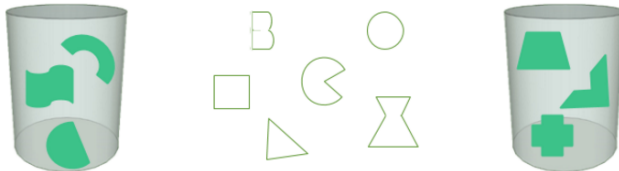
Task-Specific Questions

How did you sort the shapes in two groups?
Which attributes did you use to sort the shapes?
How could you use these attributes to name some of the shapes?
Extension: How would your grouping change if we sort these shapes into even more groups?

Students can organize items or shapes with a Venn diagram, a T-Chart, bubble map, or web.

Task

Amy's teacher asks the class to sort their shapes into two groups.



How should Amy sort her shapes based on what she observed? Explain why your sorting make sense.

Students will then be given big and small, red, yellow, and blue circles, squares, and triangles. They will then sort shapes based upon their color and size.. They will record their findings.

Share:

What attributes did you observe about these closed shapes?

What properties can you use to help you describe all polygons?

What do all of these polygons have in common?

How can you name different polygons?

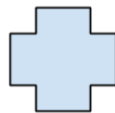
[Sharing- Venn Diagrams](#)

Summary: “We can classify two-dimensional shapes by their properties or attributes. Shapes with some attributes can grouped to make a larger category.”

Venn Diagrams- Classifying Polygons (cont.)

Launch: “I went back to my bin of buttons, and I discovered that I some triangles. How does that change my previous assortment?”

“Charlie was sorting shapes into polygons and non polygons. He placed the following shape in the polygon group. Is Charlie correct? How do you know? Explain.”



Explore: “Get with a partner and talk and share if Charlie is correct?”

Share: Students will Identify and name polygons by their attributes or properties.

Understand that there are many kinds of polygons.

Recognize the differences and similarities between polygons.

Distinguish between polygons and non-polygons.

“Now you will keep practicing in classifying polygons by comparing their properties.”

[Classifying Polygons Practice](#)

Summary: “We can classify shapes by their properties or attributes. Shapes with some attributes can grouped to make a larger category.”

Day 3: Classifying Polygons

Launch: “A group of students are helping their teacher sort some shapes. They are trying to figure out how to sort these shapes. How can they sort these shapes?”

[NCTM Lesson on Classifying Polygons with Venn Diagrams](#)

Days 4-5: Venn Diagrams- Find GCF and LCM

Launch: “Sam and Elly are making treat bags for their class party. Sam brought 32 pieces of gum. Elly brought 40 mints. What is the greatest number of equal treat bags they can make?”

“Mr. Jones orders pizza every 4 days. His neighbor, Mr. Kimble, orders pizza every 5 days. What is the least amount of days until they will both be ordering pizza on the same night?” “How can we solve each math problem?”

Probing questions:

“What is a factor? What is a multiple and what is a common multiple? What strategies could be used to make a generalization about the relationship between multiples and factors?”

What is the relationship between the least common multiple of two numbers and the factors of the two numbers?”

Explore: The goal of this task is to be able to find the greatest common factor and the least common multiple in two different situations. These questions are intended to be used with individual students or small groups as they work on the task.

General Task Questions:

What is the task asking you to do?

What information do you have?

How might you organize your work and solutions so your classmates understand it?

What might be another way to think about this problem?

Explain why your solution is reasonable.

[Venn Diagram Story](#)

Share:

What strategies could you use to find the greatest number of researchers Dr. Orion could hire so that the planets and stars are distributed equally?

What strategies could you use to determine when the two planets will be aligned again?

Are there multiple ways to solve each problem?

How are the two problems similar? How are the two problems different?

[Sharing- Finding LCM and GCF](#)

“Now let’s look at this second math problem”

Task cards 7-16.

[Learnzillion GCF and LCM](#)

Students will then practice finding the LCM and GCF

[Learnzillion Set A practice problems](#)

Share: Students will check their answers with other groups and share their findings.

Summary: “You have learned how to find the greatest common factor of two or more numbers and how to clearly explain the process for finding it. You have also learned how to find the least common multiple of two or more numbers and how to clearly explain the process for finding it.”

Day 6: Classifying Numbers

Launch: “Mr. Jones orders pizza every 4 days. His neighbor, Mr. Kimble, orders pizza every 5 days. What is the least amount of days until they will both be ordering pizza on the same night? How can we sort these numbers on a number line?”

“Let’s take a look at a this classifying numbers activity.”

Explore: Students should work in pairs to complete the Classifying Numbers Activity Sheet.

[NCTM Lesson on Classifying Numbers](#)

Share: Students will share their strategies.

Summary: Groups will present their work and explain their thinking. Students should realize that these numbers are divisible by both 2 and 3, which means they are also divisible by 6. In other words, they are all multiples of 6.

Unit 2: Area of Shapes

(Pretest) [Pre-Test Area](#)

NOTE: Throughout this unit, students **Explore** in partners or groups of four. During the **Share** portion, groups are asked to share with the class. Instructor serves as facilitator in documenting group discoveries. During the **Summarize** section, the instructor verifies that the learning goal was achieved by making statements based on discoveries.

Day 1: Area of Rectangles (backyard farm)

Launch: My backyard measures 5 yards by 5 yards. I want some farmyard pets. City law says that in order to own a bunny, each bunny needs his own space (1 yard by 1 yard). Bunnies MUST have fencing between them (or we are in trouble). I have this tool called a geoboard that will help me plan my backyard farm. It can show me the area of rectangles.

Explore: (geoboard, ipads, smartboard)

- Show (real) 5 peg geoboard and how to use it.
 - What makes a good geoboard? How big can a geoboard be?
 - Intro [digital geoboard](#) from Everyday Math e-toolkit.
- Bunnies in the yard
 - How many bunnies can I own?
 - If I just want 1 bunny, how many different places can I put his cage?
 - My Sister Sara has a backyard that measures 7 yards by 7 yards. How many bunnies can she own? (use 7-peg geoboard)
 - My Brother Ben lives in the country, and has a back yard that is 10 yards and a front yard that is 7 yards. Bunnies?
- Chickens in the yard. By law, chickens need 2 square yards.
 - (Questions similar to above, noting you can't have $\frac{1}{2}$ a chicken.)
- Goats in the yard. By law, goats need 4 square yards.
 - (Questions similar to above)

Share:

- Discuss findings as class.
- Collect data for each yard with headings: backyard size, # of bunnies.
- Add on to charts as you go, showing columns for # chickens and # of goats.

Summarize:

- The area of a rectangular shape can be found by multiplying length times width.

Day 2: Area of Rectangles (Backyard Farm cont.)
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Launch: (geoboard paper, [digital geoboard](#), centimeter paper)

Remember bunnies need 1 sq. yd., chickens need 2 sq. yds., and goats need 4 sq. yds. Using the digital geoboard and geoboard paper to record your findings, *how many ways can you make a pen* for a bunny? For a chicken? And for a goat? Work in partners.

Explore(1):

- What if I wanted a cow, which requires 16 square yards? How many ways can I build the pen?
- An elephant, which requires a square yardage of 24? How many ways can I build the pen?

Share:

- Discuss findings as they are discovered. Collect data with columns animal, required sq. footage, and # of ways to make a pen.

Explore(2):

- Pass out note cards to groups of 4, each with a number 1-20 written on it representing area. Groups will get 2-3 notecards. Group challenge is to find the most possible ways to make a rectangle with the given area.

Share:

- Construct a chart showing # of ways to make a rectangle with areas of 1, 2, 3, 4, 5, 6, 7, 8, 9, ... 20.
- Discuss findings as a class. Look for patterns.

Summarize:

- The area of a rectangle does not change if the order of length and width are switched (commutative property, i.e. $4 \times 3 = 3 \times 4$).
- There are only 2 ways to make a rectangle with an area that is prime (i.e. 1×7 , 7×1).

Day 3: Comparing the Areas and Perimeters of Rectangles

(Taken from Gr. 5 Investigations Sequence of Squares; Unit 5, Session 2.1, p.72-74)

Launch: Yesterday we imagined making some pens for different animals that required a certain amount of space, or square yards, to live in Duluth (bunny-1, chicken-2, goat-4, cow-16, and elephant-24). We discovered there was more than one way to make some of their pens. Today we are going to look at making *only square pens*. If I can only make square pens with the exact amount of space required by law, which animals can I have in Duluth? (bunny, goat, cow)

Now we are going to explore the relationship between the area of a square pen and how much fence I need to buy to make the pen.

Explore: (1-inch tiles, pg. 27 recording sheet)

- See Unit 5, Session 2.1, p. 72-74 in Gr. 5 Investigations for discussion focus points.

Share:

- Students first write answers in math journals. How much is the perimeter increasing each time? The area? Do you notice any patterns in how the area or perimeter grow?
- Can you predict what the perimeter (or area) of the next square will be?

Summarize:

- We can compare the perimeters and areas of rectangles when the dimensions are multiplied by given amounts.
- We can use numerical and/or geometric patterns to describe how the perimeters and areas of rectangles change when the dimensions change.

Day 4: Comparing the Areas and Perimeters of Rectangles (cont.)

(Taken from Gr. 5 Investigations Sequence of Squares; Unit 5, Session 2.1, p.74-75)

Launch: Yesterday we found perimeter and area for some square animal pens. We discovered that only a bunny, a goat, and a cow can have square pens if their pens were exactly the size required by the city law (bunny- 1sq. yd., goat- 4 square yds.,

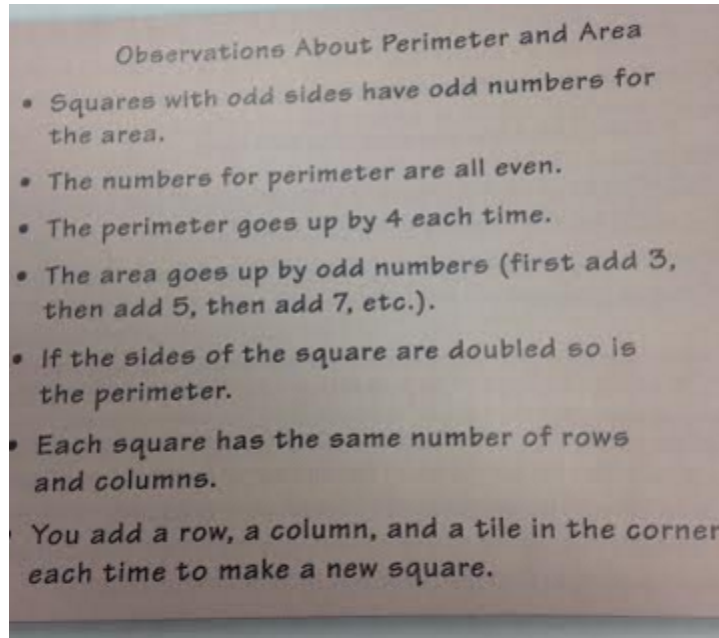
cow- 16 square yds.). If it costs \$5 per yard of fence, how much will a fence cost for a bunny? Goat? Cow?

Explore: (dot paper, geoboard paper, calculator)

- Use what you know to predict the fence cost for a square pen herd of cows, which just happens to be 100 yards by 100 yards.
- For challenge: Students work on “Area and Perimeter Fractions and Percents” page 28, Session 2.1, Unit 5, Gr. 5 Investigations.

Share:

- Students share observations from yesterday’s journal questions. Record as a class to make anchor chart that may look something like:



- Questions to think about: Why do you think that perimeter and area change in this way? Do you think it will be true for all squares?

Summarize:

- When the dimensions of the square increase by one on each side, the perimeter of the square goes up by four inches.

Day 5: Comparing Perimeters and Areas of Rectangles (contd.)

(Taken from Gr. 5 Investigations Doubling Dimensions; Unit 5, Session 2.2, p.76-82)

Launch: Knowing what we know about city law and cost of fence, what is cheaper, building pens for 4 bunnies (12 yds.) or 2 chickens (10 yds.)? 2 cows (28 yds.) or 24 goats (24 yds.) (**two animals can share one fence)

Explore:

- Students use what they observed on Day 3 & 4 and investigate what happens the sides of a rectangle are doubled.

- Explore with “Double Squares” activity sheet page 31, Session 2.2, Unit 5, Gr. 5 Investigations.

Share:

- What happens to the perimeter of a square when the side lengths are doubled? Why?

Summarize:

- When the sides of a square are doubled, the larger square has four times the area of the original square.

Day 6: Comparing Perimeters and Areas of Rectangles (contd.)

(Taken from Gr. 5 Investigations Building a Sequence of Rectangles; Unit 5, Session 2.3, p. 83-88)

Launch: Let’s go back to those animals who weren’t picky, and didn’t demand square pens. What if we had a bunch of chickens, and whenever a chicken laid a golden egg, she got an additional yard added to the length of her pen. What would the perimeter and area be after she had laid 5 golden eggs?

Explore:

- Investigate how increasing the dimensions of a rectangle affect the perimeter and area of that rectangle.
- Explore with “A Sequence of Rectangles” activity sheet pages 36-37, Session 2.3, Unit 5, Gr. 5 Investigations.

Share:

- Share results for recording sheet, make chart on board, and discuss observations.

Summarize:

- Perimeter of each rectangle increases by 14, similar to the way the perimeter of each square in on Day 5 increased by 4.

Day 7: Comparing Perimeters and Areas of Rectangles (contd.)

(Taken from Gr. 5 Investigations Building a Sequence of Rectangles; Unit 5, Session 2.3, p. 83-88)

Launch: Using what we know about our backyard farm investigations, what would be the least expensive way to build pens for 3 cows (each need

Explore:

- Explore with “Growing Rectangles” activity sheet page 38, Session 2.3, Unit 5, Gr. 5 Investigations.

Share:

- Gather data and make observations. Is the same thing happening with these rectangles as happened with the squares? Why do you think that is?

Summarize:

- Perimeter and area measure different attributes of a 2-D shape so they act differently. Perimeter is linear, or 1-dimensional, and area is 2-dimensional, and so these measurements change in different ways.

Day 8: Different Perimeter, Same Area

(Taken from Gr. 5 Investigations 2.4 Different Perimeter, Same Area; Unit 5, Session 2.4, p. 89-94)

Launch: Using what we know about our backyard farm investigations, what would be the least expensive way to build a pen for a cow, who needs 16 square feet in his pen? How much would it cost (at \$5 per yard of fence)?

We are going to look at how area and perimeter can change when the dimensions of a rectangle change. How can a rectangle maintain the same area while the perimeter changes?

Explore: (1-inch tiles)

- Students find and record area and perimeter of 4 by 6 rectangle, then “rearrange” it to form a 2 by 12 and then a 1 by 24 rectangle using “Rearranging Rectangles” activity sheet from pages 89-94, Session 2.4, Unit 5, Gr. 5 Investigations.
- Why is the area the same?

Share:

- Share observations. Create anchor chart for Same Area, Different Perimeter.

Summarize:

- We can understand square units as a unit of measure.
- We can create different rectangles with the same area but different perimeters.

Day 9: Different Perimeter, Same Area (cont.)

(Taken from Gr. 5 Investigations 2.4 Different Perimeter, Same Area; Unit 5, Session 2.4, p. 89-94)

Launch: We have been making so many different animals pens for my backyard farm. There is one more animal I have been wanting but don't have enough backyard for. It is a ferret. City law states that for each ferret I want, I need a pen that has an area of 24 square yards and a “run” that is at least 48 yards long. (Ferrets like to run.) I was thinking I could get my neighbors to go along with this idea if it is possible...would be possible to make a ferret pen on the boulevard between the sidewalk and the road?

Today we are going to be talking about how a standard unit of measure, called a square unit, doesn't always need to be a square or a rectangle.

Explore: (use dot paper, [digital geoboard](#), tiles, etc)

- Taken from “One More Cut” and “Square Units” sections of pages 89-94, Session 2.4, Unit 5, Gr. 5 Investigations.
- Can you make a shape with an area of 10 square yards and a width of $\frac{1}{3}$ yard? What is the length? What is the perimeter?
- What if I had a $\frac{1}{4}$ yard of space along the road for my ferret? Can I make a pen that is $\frac{1}{4}$ yard wide with an area of 24 square yards? How much fencing would I need?
- Can you make a shape with an area of 14 made out of a rectangle and a square put together?
- Do in partners: “More Area and Perimeter Problems” page 40, Session 2.4, Unit 5, Gr. 5 Investigations.

Share:

- Discuss answers and look for patterns.

Summarize:

- Area is the amount of space something covers. The standard unit of measure we use is a square unit, even when we are counting parts of square units.

<p>Day 10: Measuring Rectangles</p>
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(Taken from Gr. 5 Investigations Measuring Rectangles; Unit 5, Session 2.5, p. 95-99)

-Rearranging a 16 by 12 Rectangle

Launch: The difference between animals on a backyard farm and plants on a backyard farm is that plants are not picky. There is no law that says you have to have a certain amount of space to grow green beans, or carrots, or tomatoes. If I decide that the animals are not going to work out, I can always consider growing a garden. Brother Ben (the one with the big yard) has given me 12 feet of leftover fencing. If I must use whole numbers, and if my garden must be a rectangle, what different rectangles can I use to make my garden?

Explore: (1-inch tiles, centimeter grid paper, [digital geoboard](#))

- Build rectangles that have a perimeter of 30 feet. Which rectangle has the smallest area? The greatest area? Do “Fencing a Garden” activity sheet page 43, Session 2.5, Unit 5, Gr. 5 Investigations.
- Students make a succession of new rectangles by imagining the rectangle cut in half horizontally, and the two pieces put together. Do “Rearranging a 16 by 12 Rectangle” activity sheet page 44, Session 2.5, Unit 5, Gr. 5 Investigations.

Share:

- How did you find rectangles with perimeter of 30 ft.? How did you determine smallest and greatest area?
- In the Rearranging a Rectangle Activity, what did you notice about how the shape of the rectangle changed?

Summarize:

- The closer the dimensions are to each other, the closer the shape is to being a square.

- Additional Practice and/or assessment: “Garden Dimensions” activity sheet page 45, Session 2.5, Unit 5, Gr. 5 Investigations.

Day 11: Measuring Rectangles (cont.)

(Taken from BetterLesson.com, Author Carla Seeger, [Rectangles with the Same Perimeters and Different Areas](#))

Launch: (Talk about gardens....) If I have 100 feet of fencing, what are the dimensions for the largest garden I can build?

Explore:

- Students continue to investigate the relationship between area and perimeter of rectangles through partner work and independent work (both located above link). Students may use tiles, geoboards, dot paper, or centimeter grid paper if they want.

Share:

- How did you determine all of the possible dimensions?
- How did you know which dimensions would yield the greatest and smallest area?
- What units should you use? Why?

Summarize:

- A pattern can be observed in listing addends as rectangles get larger or smaller with the same perimeter and different area.

Day 12: Discovering the Formula for Triangles

(Taken from <http://illuminations/nctm.org>)

- NCTM Lesson: [Discovering the Area Formula for Triangles](#)

Launch: We have looked deeply into what to do with my backyard (talk about animals or garden, what size pen, etc.) I drive around my neighborhood and it seems everyone has a garden, or a little chicken coop, and they all look the same. Well, my daughter always wants to be different. She wants a garden that looks different than everybody elses. What if I want to make my yard look unique, and I really don't like squares or rectangles? Today we are going to consider what the square yardage of my garden might be if I use a shape other than a square or rectangle.

Explore:

- Use materials from [Discovering the Area Formula for Triangles](#)
- Use rulers to divide rectangles into triangles, discover area of triangles.

Share:

- How did you find the area of the various triangles?

Summarize:

- We can find the area of a triangle by finding a rectangle (or rectangles) around the triangle and dividing their areas in half.

Day 13: Relating a Rectangle to a Non-Rectangular Figure to find Area

(Taken from Gr. 5 Everyday Math; Vol. 2, Lesson 8-2, p. 772-777)

Launch: My neighbor has a odd shaped yard that looks like this (draw trapezoid on board with dimensions). He wants some garden and some animals. How can we make sure we follow city law with the animals and allow some area for garden? Can you come up with a plan for him?

Explore:

- Use “Focus” p. 774-777 to explore using rectangles to find the area of non-rectangular shapes.
- Students work on Math Journal 2, pg. 280 in partners.

Share:

- What are some ways to find the area of a trapezoid?

Summarize:

- We can find the area of non-rectangular shapes by visualizing a rectangle around a triangle and dividing it in half.

Day 14: Relating a Rectangle to a Non-Rectangular Figure to find area (cont.)

([Finding the Area of Triangles by Composing Parallelograms](#), taken from BetterLesson.com, Author Carla Seeger)

Launch: We moved to a new house last Christmas. The whole upstairs is carpeted, and the carpet is somewhat gross. I think the previous owners had a few dogs, and whenever I vacuum I am still (a year later) collecting dog hair. So we finally have saved enough money to buy new carpet. Carpet is sold by the square foot (draw blueprint of living room, which happens to be a trapezoid). How much carpet do I need to buy?

Explore:

- Use geoboards, dot paper, or centimeter grid paper to investigate problems in [Finding the Area of Triangles by Composing Parallelograms](#).
- Can you create a shape with an area of $15\frac{1}{2}$ sq. units on the digital geoboard?

Share:

- Share creations and how they created them.

Summarize:

- We can compose shapes to satisfy a certain number of square units.
- We can decompose shapes to determine area.

Day 15: Decomposing Shapes to find Area

(Taken from BetterLesson.com, Author Andrea Palmer)

- [Area of Composite Shapes](#)
- [Post-Test for Area of Shapes](#)

Launch: We were animals sitting my nephew's bearded dragon (Steve) in February. Steve requires a very special, very hot lamp. The special bulb gets so hot, that it will not turn back on immediately after it is shut off. Well, my daughter turned it off and set it on her carpet to give him his daily mealworms and crickets. I walked by at this point and it looked off. About ten minutes later, I started to smell something burning. The lamp had turned back on and the carpet was burning. She had apparently flipped the switch in setting it down and turned it back on (but the light did not come on). End result: large burn hole in her carpet, very sad girl. We need new carpet, etc.... (Draw room on board, how much carpet do I need?)

Explore:

- Students work together to complete "Do It Now" from [Area of Composite Shapes](#)
- Students work through the rest of the lesson, decomposing quadrilaterals to find area.

Share:

- Discuss results of [Partner Practice](#)

Summarize:

- We can decompose complex (multi-part) shapes into rectangles and triangles, find area of each, and add them together in order to find total area.
- There are often more than one way to decompose a shape, but the area will remain the same.